Proofzilla: LaTeXpackage for graphical proof theory

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CURRENT VERSION: 0.1 (unofficial release)

"I have never considered drawing as an exercise of particular dexterity, rather as principally a means of expressing intimate feelings and describing states of mind, but a means deliberately simplified so as to give simplicity and spontaneity to the expression which should speak without clumsiness, directly to the mind of the spectator."

[Henri Matisse]

"Yabadabbadoozay, baba! Bootzilla's here!"
[Bootsy Collins]

To use this LaTeX package: `\usepackage{proofzilla}`. The package is available at `https://matteoacclavio.com/Math.html?page=research#proofzilla`.

This package requires `tikz` and `txfonts` packages to work.

The package is under development, for any request/feedback/complain write me!

Contents

All the commands for drawing work using `tikz` functions `remembering position` and `overlay`. The commands for vertices and gates create an occurrence of a `tikz` node, and assign it a **nodecode** which identify its occurrence, allowing to refer to it. You can also draw edges between two vertices in the text like this edge • here •

A suffix `\vertexcode` is attached at the end of each **nodecode**. It is setted to be empty, but some times (if there are too many nodes) it can be useful to redefine it `\def\vertexcode{<newname>}` to avid that `tikz` mixes **nodecode**s. You can ignore its existence most of the time.

1 Symbols

The following symbols are defined:

`\ltens` = ⊗	`\lpar` = ⅋	`\lone` = 1	`\lbot` = ⊥
`\lwith` = &	`\lplus` = ⊕	`\ltop` = ⊤	`\lzero` = 0
`\oc` = !	`\wn` = ?	`\ldia` = ◇	`\lbox` = □
`\limp` = ⊸			

(1)

If some of these commands are already defined by some other command/package, they will not be redefined. Moreover, the package uses the symbols from the package `cmll` if provided.

2 Graphs

To represent graphs, use the environment `array` to have a virtual grid to place vertices on/in it.

2.1 Vertices

The package provides two commands to define two types of vertices as nodes in `tikz`:

- `\newvertex{<name>}{<label>}{<options>}` defines the command

  `\v<name>{<occurenceId>}`

  for fixed-labelled `<label>` vertices/nodes.

- `\newemptyvertex{<name>}{<options>}` defines the command

  `\v<name>{<occurenceId>}{<label>}`

  for vertices witch label `<label>` can be specified.

Each occurrence of both commands generates a vertex/node with associated **nodecode** `<name><occurenceId>`. Use `<options>` to provides additional options to the command `node[<options>]`.

Some examples:

```
\newvertex{name}{label}{}
\newvertex{square}{sq}{draw,circle}
\newemptyvertex{module}{draw}
```

\vname1 = *label*
\vsquare1 = (*sq*)
\vmodule1{foo} = foo

The labels of graph vertices are defined in `$math$` environment.

2.2 Edges

The package provides a command to define edges styles.

```
\defedgetype{<name>}{<draw options>}{<to options>}
```

To understand the options, think that the edges of that type are drawn in `tikz` using

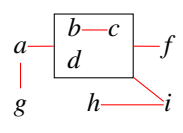
```
\draw[<draw options>] (<source>) to [<to options>] (<target>)
```

Each call of `\defedgetype` defines the following commands:

- `\<name>edge{<source>}{<target>}` draws an edge of type `<name>` from node with **nodecode** `<source>` to node with **nodecode** `<target>`;
- `\<name>edges{<list>}` draws an edge of type `<name>` for each pair of the list `<list>` in the form `{source/target,...}` from each source to each target;
- `\multi<name>edge{<list1>}{<list2>}` draws an edge of type `<name>` from each node in `<list1>` to each node in `<list2>`.
- `\bent<name>edges{<list>}` draws an edge of type `<name>` for each triple of the list `<list>` in the form `{source1/target1/bendvalue1,...}` from each source each target with the corresponding `bend left` value;
- `\spec<name>edge{<source>}{<target>}{<to options>}` draws an edge of type `<name>` from from node with **nodecode** `<source>` to node with **nodecode** `<target>` with additional `to [<to options>]` options ... Just because some time you need a special edge;
- `\<name>ledge{<source>}{<target>}{<label>}` draws an edge of type `<name>` from node with **nodecode** `<source>` to node with **nodecode** `<target>` with a label `<label>` (midway).
- `\<name>ledges{<list>}` draws a labelled edge of type `<name>` for each tripe in the list `<list>` in the form `{source/target/label,...}`;
- `\<name>sameledges{<list>}{<label>}` draws a labelled edge of type `<name>` for each pair in the list `<list>` in the form `{source/target,...}`; all with the same label `<label>`;

- \spec<name>ledge{<source>}{<target>}{<to options>}{<label>} draws a labelled edge of type <name> from from node with **nodecode** <source> to node with **nodecode** <target> with additional to [<to options>] options and label <label>.

```
\begin{array}{ccc} \va1 &
\vM1{\begin{array}{cc}\vb1&\vc1\\\vd1\end{array}}
&\vf1\\[1em] \vg1 &\vh1 & \vi1 \end{array}
\testedges{a1/M1,M1/f1,b1/c1,g1/a1,i1/h1,i1/M1}
```



Moreover, \defedgetype also define the following commands for edges with a <label> marked in the midway of the edge

- \<name>ledge{<source>}{<target>}{<label>} draws an edge of type <name> from **nodecode** <source> to <target> with label <label>;
- \<name>ledges{<list>} draws a labelled edge for each triple of the list <list> in the form {source/target/label,...};
- \<name>sameledges{<list>}{<label>} draws a labelled edge for each pair of the list <list> in the form {source/target,...}, all with the same label <label>;
- \spec<name>ledge{<source>}{<target>}{<to options>}{<label>} same as \spec<name>edge but with an additional argument for the label.

3 Combinatorial proofs

The package provides the definition of logic negation \cneg<arg> as \bar<arg> if not already defined.

The following commands for vertices are pre-defined using \newvertex: atomic variables, i.e. are lowercase alphabetic letters \va#1...\vz#1, with their negation \vna#1...\vnz#1 and the following ones[1]

\vlone#1 = 1 \vlbot#1 = ⊥ \vltop#1 = ⊤ \vlzero#1 = 0
\voc#1 = ! \vwn#1 = ? \vlbox#1 = □ \vldia#1 = ◇
\vjump#1 = ∘

Their **nodecode** is given by removing the letter v from the command name, e.g., the **nodecode** of the vertex \vlbot7 is lbot7.

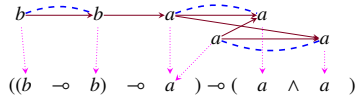
[1] Note that redefining commands in Equation (1) will change labels accordingly.

The following standard edge types for combinatorial proofs are provided:

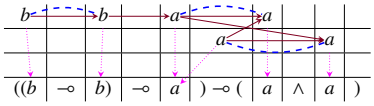
`D` or `dirgraph`	=		`G` or `graph`	=	
`DR` or `dircograph`	=		`R` or `cograph`	=	
`A` or `arena`	=				
`M` or `mod`	=		`L` or `link`	=	
`S` or `skew`	=		`B` or `matching`	=	
`doubleS`	=		`doubleB`	=	

The command `\cutshade<south-west><north-east>` draws a shaded (in grey) rectangle with south-west corner the vertex `<south-west>` and north-east corner the vertex `<north-east>`. The command `\vhid#1` is provided for a vertex with no labels and with **nodecode** `hid#1`. It can be used in case there are no vertices in the corners of the desired `cutshade`.

```
\begin{array}{cccccccc}
\vb1&&\vb0&&\va1&&\va0\\
\voc1&&\vwn1&&\voc2&&\vwn2\\
&&&&&\va3&&&\va2\\ \\
((\vb5&\limp&\vb4)&\limp&\va7&)
\limp(&\va4&\land&\va6&)
\end{array}
\Bedges{a1/na1,a2/na2}
\dmatchingedges{oc1/wn1,wn2/oc2}
\multiRedges{na1,wn1}{a2,oc2}
\Medges{oc1/a1,wn1/na1,oc2/a2,wn2/na2}
\Sedges{a1/a3,na1/na3,a2/a2,na2/na4}
\Aedges{b1/b0,b0/a1}
\multiAedges{a1,a3}{a0,a2}
\Sedges{b1/b5,b0/b4,a1/a7,a3/a7,a0/a4,a2/a6}
\bentLedges{a2/a3/20,a1/a0/20,b1/b0/20}
```



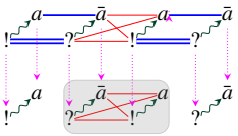
The corresponding combinatorial proof



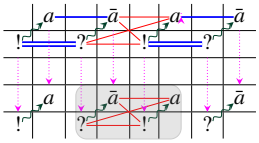
The corresponding combinatorial proof showing the underlying grid of the array

Figure 1: An intuitionistic combinatorial proof

```
\begin{array}{cccccccc}
&\va1&&\vna1&&\va2&&\vna2\\
\voc1&&\vwn1&&\voc2&&\vwn2\\ \\
&\va3&&\vna3&&\va4&&\vna4\\
\voc3&&\vwn3&&\voc4&&\vwn4\\
\end{array}
\Bedges{a1/na1,a2/na2}
\dmatchingedges{oc1/wn1,wn2/oc2}
\multiRedges{na1,wn1}{a2,oc2}
\Medges{oc1/a1,wn1/na1,oc2/a2,wn2/na2}
\Sedges{a1/a3,na1/na3,a2/a2,na2/na4}
\Sedges{oc1/oc3,wn1/wn3,oc2/oc4,wn2/wn4}
\multiRedges{wn3,na3}{oc4,a4}
\Medges{oc3/a3,wn3/na3,oc4/a4,wn4/na4}
\cutshade{wn3}{a4}
```



The corresponding combinatorial proof



The corresponding combinatorial proof showing the underlying grid of the array

Figure 2: A combinatorial proof with cuts

4 Interaction nets

As for graphs, use the environment `array` to have a virtual grid to place gates on/in it.

4.1 Gates, inputs and outputs

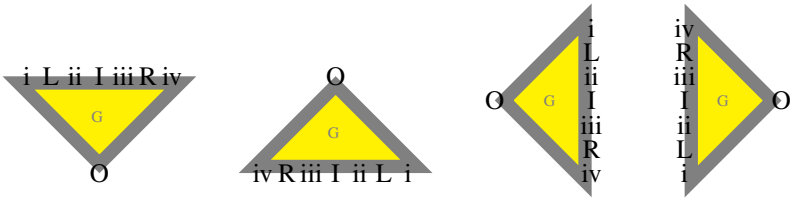
The package provides a command to define proof structures gates:

```
\newgate{<name>}{<label>}{<options>}
```

Each command provides the following commands to draw gates (where `<label>`= *X*):

command	**nodecode**	node representation
`\G<name>{<occId>}`	G<name><occId>	
`\uG<name>{<occId>}`	uG<name><occId>	
`\lG<name>{<occId>}`	lG<name><occId>	
`\rG<name>{<occId>}`	rG<name><occId>	

Gates have the `isosceles triangle`'s anchors plus the following:



(2)

4.1.1 Inputs and outputs

The package also provides commands to define input/outputs or floating labels

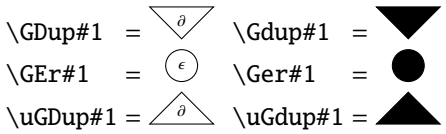
```
\psnode[{<optional-label>}]{<occurrenceId>}
\psanode[{<optional-label>}]{<occurrenceId>}
\pslnode{<label>}{<occurrenceId>}
\pshang{<occurrenceId>}
```

which respectively produce nodes with **nodecode**s `node<optional-label><occurrenceId>`, `node<optional-label><occurrenceId>`, `node<occurrenceId>`, and `hang<occurrenceId>`. To remember the commands: *a* stands for *anonym* and *l* stands for *labelled*.

command	**nodecode**	node representation
\psnode[a]2	nodea2	$_a$
\psnode 1	node1	
\pslnode a 2	nodea2	$_a$
\psanode[a]2	node2	$_a$
\psanode 3	node3	
\pshang 1	hang1	$_\circ$

Nodes generated by these commands have standard rectangle anchors plus `I` (north) and `O` (south) and `C` (center).

The following commands for gates provided:



4.2 Wires

The package provides a command to draw a wires:

- `\pswire{<source>}{<target>}{<looseness>}` draws a single (unlabelled) wire from an input to an output;
- `\pslwire{<source>}{<target>}{<looseness>}{<label>}` draws a single labelled wire;
- `\pswires{<list>}` draws wires from a list `{element,...}` of with elements of form `source/target` or `source/target/label`;
- `\psbentwires{<list>}` draws wires with specified `looseness` for a list with elements of either forms

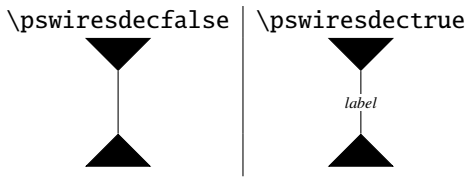
  `source/target/looseness` or `source/target/label/looseness`

If only the **nodecode** of a gate is given, then the wire come out/in from its `center` anchor. Use the anchors in Equation (2) to specify where the wire is attached, e.g., `G<name><occurrence>.<anchor>`.

Wires comes in and out of a gate at an angle of respectively `90` and `-90` degree (`\topdownps`). If proof structures are represented horizontally (from left to right), you can change these angle to respectively `180` and `0` degree using the command `\lefttorightps`.

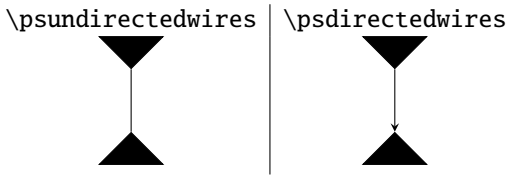
4.2.1 Labels on wires

Wires labels are in `$math$` environment. By default `\pswiresdecfalse`, that is, wires are unlabelled. It is possible to reveal/hide wires label respectively using `\pswiresdectrue` and `\pswiresdecfalse`.



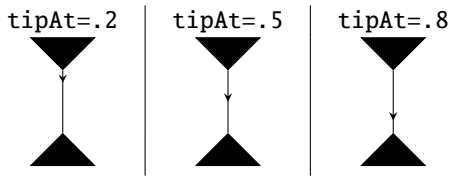
4.2.2 Orienting wires

By default proof structure wires are non-oriented. Use the commands `\psdirectedwires` and `\psundirectedwires` to respectively enable and disable wires orientation.



Additional commands to draw wires arrow tip in a specific position are provided.

- `\psowire{<source>}{<target>}{<looseness>}{<tipAt>}` draws a wire from <source> to <target> with a given <looseness> and arrow tip in position <tipAt>;
- `\psowires{<list>}` draws wires from a list `{element,...}` of with elements of form `source/target/looseness/tip-position`.



These commands do not support wire labels.

4.2.3 Axioms and Cuts

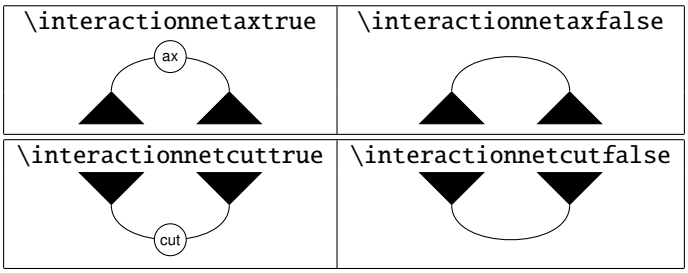
The package provides the following commands to draw for axioms:

- `\psaxiom{<target1>}{<target2>}{<looseness>}` draws a wire from the gate with **nodecode** <target1> to node with **nodecode** <target2> with looseness value <looseness>;
- `\psaxioms{<list>}` draws an axiom for each pair of the list <list> of the form `{target1/target2,...}`;
- `\psbentaxioms{<list>}` draws an axiom with given looseness for each triple of the list <list> in the form `{target1/target2/looseness,...}`.

Similar commands are defined for cuts.

```
\pscut{<target1>}{<target2>}{<looseness>}
\pscuts{<list>}
\psbentcuts{<list>}
```

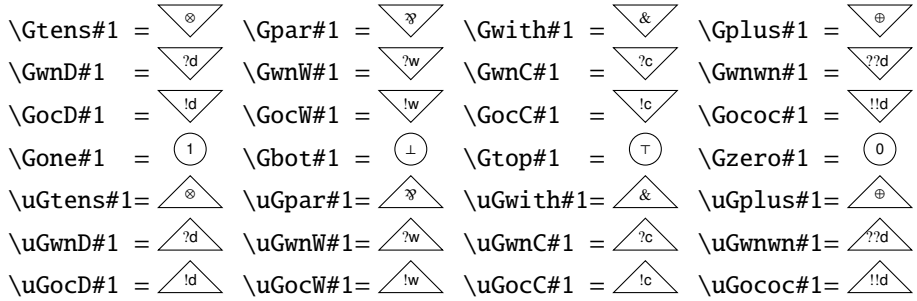
By default proof structures are represented in interaction nets syntax, that is, axioms and cuts are wires. It is possible to enable the explicit representations of axioms using `\interactionnetaxtrue` and cuts using `\interactionnetcuttrue`.



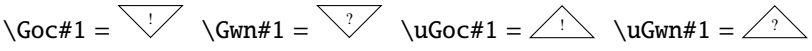
The labels for axiom and cut gates are respectively ax and cut. It is possible change these labels using `\changeaxsymbol<newsymbol>` and `\changecursymbol<newsymbol>`.

4.3 Linear Logic Proof Structures

The following commands for gates for standard connectives are provided:



Plus the following ! and ? generic gates

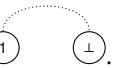


4.3.1 Jumps

The package provides the following command to draw jump edges (similar to the ones for axioms/cuts):

- `\psjump{<target1>}{<target2>}{<looseness>}` draws a jump edge between `<target1>` and `<target2>` with given `<looseness>`;

- `\psjumps{<list>}` draws a jump edge for each pair in the `<list>` of the form `{taget1/target2,..}`
- `\psbentjumps{<list>}` draws a jump edge for each triple in the `<list>` of the form `{taget1/target2/looseness,..}`

For example `\Gone1\qquad\Gbot1\psjump{Gone1.I}{Gbot1.I}{}` gives 

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It is possible to change the style of jumps wires using the command

```
\changejumpstyle{<tikz options>}
```

4.3.2 Boxes

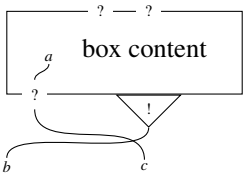
Linear logic boxes are defined by positioning two vertices `\boxYin{<boxId>}` and `\boxYang{<boxId>}` and then calling the command

```
\psBox[<orientation>]{<boxId>}{<principalanchor>}{<list>}
```

which draws a box as follows:

- it draws a rectangle with corner `\boxYin{<boxId>}` and `\boxYang{<boxId>}`;
- it place an !-gate with **nodecode** `box<boxId>main` at the anchor `<principalanchor>` of the rectangle. If `<orientation>` is not given or if it is `D`, the gate points downwards, if it is `U` the gate points upwards.
- for each `<anchor>` in `<list>`, it draws an auxiliary port, that is a `psnode`, on the anchor `<anchor>`. Each auxiliary port has **nodecode** `\box<boxId>aux<anchor>`.

```
\begin{array}{ccccc}
\boxYin1\\
&\pslnode a1&\mbox{box content}&\\[.5em]
&&&\boxYang1\\[1em]
\pslnode b1 &&\pslnode c1\end{array}
\psBox{1}{-60}{-155,60,120}
\pswires{nodea1/box1aux-155,box1aux-155/nodec1}
\psbentwires{box1main.O/nodeb1/.6}
```



Acknowledgements

Thanks to Lutz Straßbourger to have shared his macros for vertices and edges from which the package has evolved to the current shape.