AI504 Knowledge Representation
Exercises 2

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1. Consider the following sentence:

$$((Food \rightarrow Party) \vee (Drinks \rightarrow Party)) \rightarrow ((Food \wedge Drinks) \rightarrow Party)\,.$$

   (a) Determine, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

   (b) Convert the left- and righthandsides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

   (c) Prove your answer to (a) using resolution.

2. Using resolution prove that

   (a) $A \vee B, A \rightarrow C, B \rightarrow C \vdash C$

   (b) $A, \neg A \vdash B$

   (c) $(A \rightarrow B) \rightarrow B, (A \rightarrow A) \rightarrow C, (C \rightarrow D) \rightarrow \neg(D \rightarrow B) \vdash C$

   (d) $A \rightarrow B, \neg A \rightarrow C \vdash \neg B \rightarrow C$

   (e) $(A \rightarrow B) \vee (C \rightarrow D) \vdash (A \rightarrow D) \vee (C \rightarrow B)$

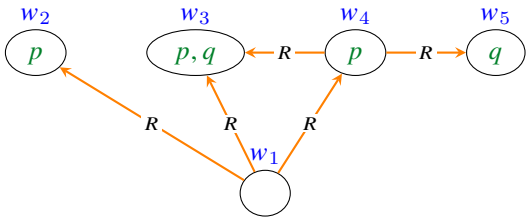
   (f) $\neg(A \wedge \neg B) \vee \neg(\neg C \wedge \neg D), \neg(D \vee B), E \rightarrow (\neg D \rightarrow (\neg C \wedge A)) \vdash \neg E$

3. Consider the following information.

   - If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.
   - If the unicorn is immortal or a mammal, then it is horned.
   - The unicorn is magical if it is horned.

   Can you prove that the unicorn is mythical? How about magical? Horned?

4. Consider the following model:



   Decide if the following sentences are true or false in the model:

   (1) $w_1 \vDash \Box p$
   (2) $w_1 \vDash \Box q$
   (3) $w_1 \vDash \Diamond q$
   (4) $w_4 \vDash \Box q$
   (5) $w_2 \vDash p \wedge \Box p$
   (6) $w_3 \vDash p \wedge (\Diamond p \rightarrow q)$
   (7) $w_1 \vDash \Diamond \Box q$
   (8) $w_1 \vDash \Box\Box q$
   (9) $\vDash \Diamond p$
   (10) $\vDash \Box\Box q$
   (11) $\vDash \Box(q \vee \Box q)$
   (12) $\vDash (p \vee \Diamond p) \rightarrow q$

5. Define a model M (respectively N) with at least three worlds which satsifies (respectively which does not satisfy) the following formulas:

   (a) $\Box p \to \Diamond p$
   (b) $\Box p \to \Box\Diamond p$
   (c) $\Diamond p \to \Box\Diamond p$
   (d) $\Box p \wedge \Diamond\Box p$
   (e) $\Diamond p \wedge \Diamond\Box p$
   (f) $\Box p \wedge \Box\Box p$
   (g) $\Diamond p \vee \Diamond\Diamond p$
   (h) $\Box p \vee \Box\Box p$
   (i) $\Diamond p \vee \Diamond\Diamond p$

6. See exercise 4 in Sheet 3.